Remote Sensing

Topic 8: Vertical Measurements

Chapter 3: Lillesand and Keifer
Chapter 7: Paine

Options: Single Photo

1. Topographic or Radial Displacement

2. Shadow Method
   - Sun Angle Method
   - Proportional Shadow Length Method

Options: Stereopair

1. Parallax Height Determination

Topographic Displacement

- Recall, displacement was a function of:
  - radial distance of an object from the nadir
  - height of the object
  - elevation of the aircraft above the datum

$d = r\,h / H$ or $h = d\,H / r$

Where: d = displacement h = height of object
r = distance from nadir to top of displaced object
H = height of aircraft above datum, also (A-E)

- Provided H is known or can be determined, d and r can be measured on the photography

Topographic Displacement

- Conditions for measuring height:
  1.
  2.
  3.
  4.
  5.

Shadow Method

- Two variations:
  - __________________ method requires that:
    1. Height of an object casting a measurable shadow is known
    2. Object of interest is casting a measurable shadow
  - If no known ht., ___________ method may be used IF:
    1. Latitude of object is known
    2. Time of day can be determined
    3. Day of year is known

Proportional Shadow Method

- Ratio of height to shadow length is constant for all features at a given Sun angle

$$H_1/SL_1 = H_2/SL_2$$

So if H_1 is unknown, but H_2 is known
And SL_1 and SL_2 can be measured:

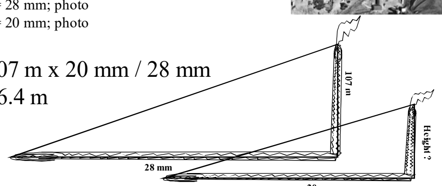
$$H_u = H_k SL_u / SL_k$$

Proportional Shadow Method

Example 1:
$H_u = H_k SL_u / SL_k$

H_k = 107 m; from topo sheet
SL_k = 28 mm; photo
SL_u = 20 mm; photo

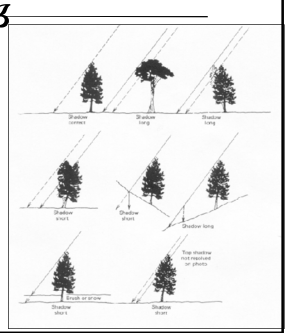
H_u = 107 m x 20 mm / 28 mm
H_u = 76.4 m



Accurate Shadow Length

Shadow length may be inaccurate when:

1.
2.
3.
4.



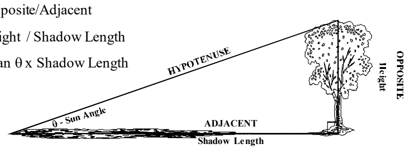
Sun Angle Shadow Method

- Height of object is determined through trigonometry
- Provided the length of the shadow can be measured and angle of incidence of Sun is known

Tan θ = Opposite/Adjacent

Tan θ = Height / Shadow Length

Height = Tan θ x Shadow Length



Sun Angle Shadow Method

- To determine Sun angle requires:
  - latitude
  - time of day
  - and day of the year

$$\sin\theta = (\cos x)(\cos y)(\cos z) \ +/- \ (\sin x)(\sin y)$$

where: x = Sun's declination on day of photo (**from analemma**)
y = latitude of location in decimal degrees **(from map)**
z = hour angle, difference in degrees of longitude between local solar time and sun noon
= ΔT (Sun Noon and LST in decimal hrs.) x 15°/hr
\+ if location and declination are in same hemisphere
\- if in opposite hemispheres

Example 1: Sun Angle

- Scale - 1:6000
- Date - April 24, 2001
- Time of Day – 10 am DST
- Latitude – 49 º 51"
- Determine ht.. of the largest chimney

Example 1: Sun Angle Method

x = Sun's declination on the day of photography.

From analemma (next slide) declination on April 24 is = 12.6 º

y = latitude of location in decimal degrees

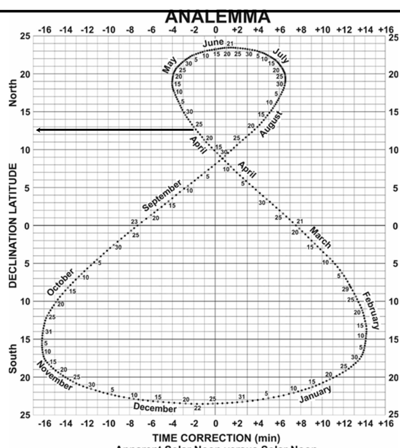
Latitude = approx. 49 º 51" so y = 49.85 º

z = hour angle, difference in degrees of longitude between local solar time and sun noon.

Time of day is 10:00 am DST or 9:00 am LST

Hour angle = |12:00 – **9:00**| = 3:00 or 3.0 hrs x 15 º = 45º = z

Note: 9 am is approximate solar time, doesn't account for ET or objects location within standard time meridian



Example 1: Sun Angle Method

$\sin\theta = (\cos x)(\cos y)(\cos z)$ + or - $(\sin x)(\sin y)$

$\sin\theta = (\cos 12.6^\circ)(\cos 49.84^\circ)(\cos 45^\circ)$ + or - $(\sin 12.6^\circ)(\sin 49.84^\circ)$

$\sin\theta = (0.445) + (0.167)$

$\sin\theta = 0.612$

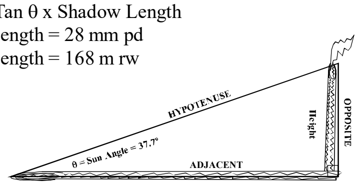
$\theta = 37.7^\circ$

Height = Tan θ x Shadow Length

Shadow Length = 28 mm pd

Shadow Length = 168 m rw

$\theta = 37.7^\circ$



Height = Tan 37.7º x 168 m

Height = 0.773 x 168 m

Height = 129 m

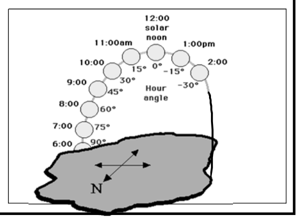
Adjusted Solar Time

- Need accurate solar time to derive hour angle

LST = LT + [4 minutes (LL - LSTM) + ET]

where : LST = Solar Time
LT = Local Time
LL = Local Longitude
LSTM = Local Std Time Meridian
ET = Equation of Time (analema)

Note: Keep track of +/- signs.



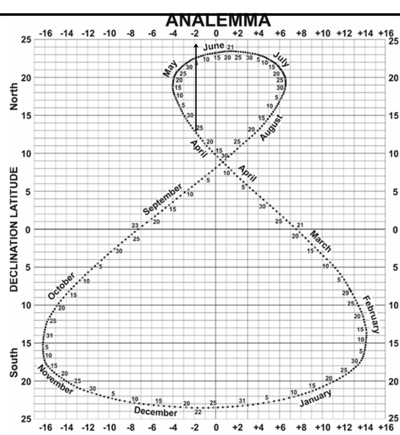
Adjusted Solar Time

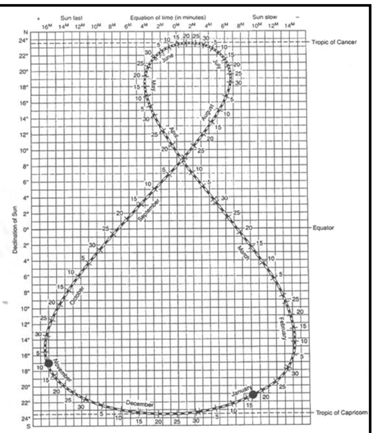
- Need accurate solar time to derive hour angle

LST = LT + [4 min. (LL - LSTM) + ET]
= 9:00 am + [4 min. (-99.89° - -90°) - 1.8 min.]
= 9:00 am + [4 min. (-9.89°) - 1.8 min.]
= 9:00 am – [39.56 min. - 1.8 min.]
= 9:00 am – 41.~~36~~ min
= 8:19 am is adjusted solar time

LST = Solar Time
LT = 9:00 am
LL = 99° 53.381"W
= -99.89°
LSTM = -90°
ET = 1.5 min.

Hour angle = |12 – 8:19 am| = 3:41 or 3.68 hours
3.68 hours x 15°/hr = **55.2°**





Example 2: Sun Angle Method

using adjusted solar time and hour angle

$\sin\theta = (\cos x)(\cos y)(\cos z) + \text{or} - (\sin x)(\sin y)$

$\sin\theta = (\cos 12.6^\circ)(\cos 49.84^\circ)(\cos 55.2^\circ) + \text{or} - (\sin 12.6^\circ)(\sin 49.84^\circ)$

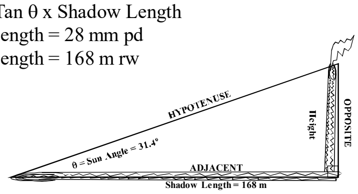
$\sin\theta = (0.359) + (0.167)$

$\sin\theta = 0.526$

$\theta = 31.7^\circ$

Height = Tan θ x Shadow Length
Shadow Length = 28 mm pd
Shadow Length = 168 m rw
θ = 31.7º

Height = Tan 31.7º x 168 m
Height = 0.617 x 168 m
Height = 103.7 m



Parallax

- Refers to apparent shift in position of an object
- Result of change in the position of observation
- Amount of shift dependent on:
  - distance of object from the camera
  - or height of the object
- Objects at same elevation have same amount of shift

Parallax Height Determination

- Typically most accurate method of determining the absolute or relative height of objects
- Requires:
  1.
  2.
  3.

Derivation of Parallax Height Equation

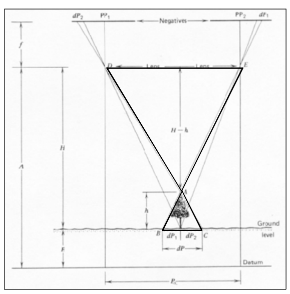
Δ ABC = Δ ADE

Therefore:

h / dP = H-h / P

or

h = dP(H) / P + dP



Parallax Height Determination

h = dP(H) / P + dP

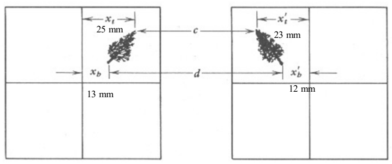
Where: dP = absolute difference of parallax between top and bottom of object <u>measured off photos</u>

H = height of aircraft above datum

P = absolute parallax at the base of the object; measured as the average distance between PP and CPP on both photos

- Note: level terrain eq. valid when $\pm\Delta E$ is < 5% H
  - At 20,000 ft. $\pm\Delta E$ could be as much as 1000 ft.

Parallax Measurement

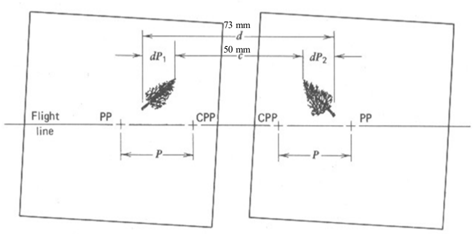


Absolute parallax of top = $x_t - (-x'_t) = x_t + x'_t$ 48 mm

Absolute parallax of bottom = $x_b - (-x'_b) = x_b + x'_b$ 25 mm

Difference in absolute parallax, $dP = (x_t + x'_t) - (x_b + x'_b)$ 23 mm

Parallax Measurement



$dP = x_t - x_b + x'_t - x'_b = d\text{-}c = dP_1 + dP_2$

Parallax Procedure

1. Locate PP and CPP; identify flight line
2. Orientate and secure stereo model
3. Determine average photo base by measuring distance between PP and CPP on each photo **(P)**
4. Determine absolute parallax of top of object using parallax bar (or ruler)
5. Repeat step 4 for the bottom of the object
6. Calculate difference in absolute parallax between the top and bottom of the object **(dP)**
7. Determine height of the aircraft above datum **(H)**
8. Use parallax height equation