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Decision Support

The influence of coupon duration on consumers' redemption behavior and brand profitability

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ABSTRACT

This research proposes an analytical model of the joint optimization of coupon face value and duration together with the product price, and determines the impact of coupon design on consumers' redemption behavior. A model of rational forward-looking consumers' redemption behavior is derived that incorporates forgetting (to redeem) and stochastic redemption costs.

Results show that when product price is exogenous, long-duration coupons may result in increased seller profits and always increase consumer surplus. Moreover, a one-period coupon is never optimal when (1) the difference in valuations for high-value (loyal) and low-value (non-loyal) consumers or (2) the coupon face value is larger than the redemption costs of high-value consumers. Long-duration coupons tend to be optimal when the level of recall of high-value consumers is sufficiently low, which reduces redemption by high-value consumers.

Coupon duration together with face value plays an important role in coupons' ability to price discriminate between different consumer segments and to avoid head-on competition with other sellers. Results can replicate empirically observed redemptions patterns, which has important implications for the strategic targeting of coupons to different consumer segments.

A coupon may result in an increase or decrease in price. When the difference in valuation between high-value and low-value consumers is high (relative to the redemption costs), a seller can either reduce price and lower face value or increase coupon duration for the purpose of avoiding redemption by high-value consumers.

1. Introduction and background

About 244 billion coupons were distributed throughout the US in 2018 with a total face value of $497 billion Kantar Media (2018). These promotional strategies have an important influence on the profitability of retailers (Mou, Robb & DeHoratius, 2018). Manufacturers and retailers use coupons to stimulate short-run sales and new-product trial (Freimer & Horsky, 2008; Johnson, Tellis & Ip, 2013). In mature markets, coupons are used to induce switching to the promoted product, resulting in a temporary increase in sales (Hübner, Kuhn & Kühn, 2016; Kogan & Herbon, 2008; Martín-Herrán & Sigué, 2011).

To implement a coupon strategy, a manufacturer or retailer needs to determine the coupon's (1) face value and (2) duration. The face value is the amount of discount offered to the consumer. In addition, the coupon has a certain expiration date before which the coupon must be used. We call the time until the expiration date the coupon duration.

Although previous research has studied coupon face value and coupon duration in isolation (see literature review), no previous research has considered the dependence of the optimal face value and coupon duration. Because a coupon strategy consists of these two decision variables, considering them jointly is essential. The face value has important implications for sellers' profitability through the number and types of consumers attracted by coupons. In selecting the optimal face value, the sellers need to trade-off the gain from attracting new consumers with the loss from coupon redemption by loyal consumers. Coupon duration also has important implications for sellers' profitability through the redemption patterns and the number of coupons redeemed.

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Therefore, the main objective of this research is to study the joint optimization of coupon face value and duration. Also, the optimal coupon strategy may be determined together with the regular pricing strategy (e.g., a manufacturer may use a higher regular price in combination with price discounts through coupons). As a result, we consider both a model in which price is fixed (Model 1 with exogenous price) and a model in which price is allowed to vary as a decision variable (Model 2 with endogenous price). In addition, we study how different coupon designs (face value and duration) influence consumers' coupon-redemption behavior. Finally, we extend our model to a duopoly scenario with two competing sellers who use coupons to attract the non-loyal or low-value consumers.

We propose a model that analytically derives optimal coupon face value and duration, assuming both exogenous and endogenous pricing decisions. Consumers are assumed to be rational and *forward-looking*, and the model allows for *forgetting* and *stochastic redemption costs*, adding richness to our conceptualization. The term *forward-looking* consumers implies strategic consumers who will consider future purchasing conditions in determining the best time to redeem their coupon (e.g., a shopping trip when they are not rushed). *Forgetting* is a fundamental and pervasive aspect of almost all redemption behavior, because consumers may forget to redeem their coupon (Silk, 2008; Su, Zheng & Sun, 2014). *Stochastic redemption costs* take into account that the cost of redeeming a coupon (the physical effort of going to the retailer's location as well as the transaction costs—extract the right coupons, organize them, and bring them to the store to obtain the discount), tend to vary over time, as consumers may have different time constraints for various shopping trips (Chen, Moorthy & Zhang, 2005; Chun, 2012; O'Donoghue & Rabin, 1999). Although forgetting (Su et al., 2014) and stochastic coupon redemption (Chen et al., 2005; Chun, 2012) have been studied before, none of these papers have included coupon-duration strategies.

We focus on traditional coupons because the majority of coupons are traditional coupons mass distributed to consumers. According to recent statistics, of the 244 billion coupons distributed in 2018, 94% were distributed through free-standing inserts (FSI) and only 1.5% were digital coupons.[1] Furthermore, traditional print coupons have eight times the redemption rate of digital coupons (News America Marketing, 2015). However, in recent years, targeted and digital coupons are increasingly used by retailers (Li, Liou & Ni, 2019; Sahni, Zou & Chintagunta, 2016; Shaffer & Zhang, 2002). Hence, although the current focus is on traditional coupons, future research may want to consider targeted coupons.

We find the joint optimization of coupons' face value and duration provide several important and novel insights related to (1) optimal face value and duration, (2) coupons as a price-discrimination device, and (3) coupon-redemption patterns.

Optimal coupon face value and duration. The optimal face value and corresponding payoffs for coupons with different durations and redemption patterns can be determined by using Tables 4 and Appendix C (for exogenous prices) and Tables 7 and 8 (for endogenous prices).

A longer-duration coupon will result in a gain from increased redemption by low-value consumers, but also a loss due to increased redemption by loyal or high-value consumers. Therefore, a seller needs to trade off these potential gains and losses. A longer-duration coupon is optimal when the level of recall of high-value consumers is sufficiently low. When price is exogenous, a one-period coupon is never optimal when (1) the difference in valuations for the high-value (loyal) and the low-value (non-loyal) consumers or (2) the coupon face value is larger than the high redemption costs of the high-value consumers. Consumer surplus is always higher with a longer coupon duration, because consumers benefit from longer redemption windows that provide them with greater flexibility in determining the best time to redeem (i.e., lower redemption cost), even though redemption rates tend to be lower due to forgetting.

Moreover, for an endogenous price, we show that when the difference in valuation between high- and low-value consumers is high (relative to the redemption costs), a seller can either reduce the price and lower the face value or increase the coupon duration to avoid redemption by high-value consumers.

Coupons as a price-discrimination device. Our results show that coupon duration (together with face value), through consumers' redemption behavior, plays an important role in coupons' ability to serve as an effective price-discrimination device between low- and high-value consumers. These results extend previous research, which has shown that coupons can be used as a strategic tool to price discriminate among consumers with different levels of price elasticity (Dhar & Hoch, 1996; Narasimhan, 1984).

Coupon-redemption patterns. Face value and duration play an important role in consumers' redemption behavior. We show that previous empirical findings, attributed to irrational consumer behavior such as regret and/or procrastination, are consistent with assumptions of rational forward-looking consumers (implying consumers have a correct perception of the future). In particular, we find that different combinations of face values and durations can explain empirically observed redemption patterns, including a monotonic decreasing trend, where most coupons are redeemed early on and redemption reduces over time (e.g., Neslin, 1990), and a decreasing trend with a spike close to expiration (Inman & McAlister, 1994).

Finally, we extend our model to a duopoly in which two sellers compete. Both sellers determine the face value and duration of their coupons, competing for the low-value or non-loyal consumers, while considering the potential loss from their loyal consumers. We derive equilibrium conditions and find conditions under which different coupon strategies are optimal. We find that sellers can avoid head-on competition, by alternating the time when they offer a coupon, when a one-period coupon is optimal for at least one of the sellers. When using a long-duration (three-period) coupon is optimal for both sellers, either both sellers will compete or one seller will use a suboptimal strategy (short-duration coupon) to avoid competition.

These results have important implications for managers in determining optimal coupon strategies. First, we provide managers with conditions under which they should use different coupon durations to determine the optimal coupon face value and profits. Second, we provide guidelines for pricing strategies combined with coupon design. In particular, we indicate instances in which sellers are better off reducing the price and offering a coupon with a lower face value or increasing coupon duration to avoid redemption by high-value consumers. Third, we show that coupon duration and face value can be used as strategic tools to price discriminate between different consumer segments and to avoid head-on competition with other sellers. Finally, our findings that empirically observed redemptions patterns can be derived assuming rational forward-looking consumers has important implications for the strategic targeting of coupons to different consumer segments.

The remainder of the paper is organized as follows. In the next section, we review the related literature and position our contributions. In the third and fourth sections, we formally investigate the coupon-design decisions including the coupon's face value and duration, while incorporating the consumers' redemption behaviors when the shelf price is either exogenous or endogenous. In the fifth section, we extend our model to a duopoly

[1] Source: NCH U.S. CPG Coupon Facts: Year-end 2018, Available at https://www.nchmarketing.com/couponindustrytrends.aspx. Assessed March 22, 2019.

environment where competing firms issue coupons. Finally, we conclude by summarizing contributions and implications.

2. Literature review

Given the importance of coupons as a tool to attract consumers, the proper design and strategy have been focal questions for both scholars and practitioners. We discuss theoretical research on coupon design that has focused on the face value and the duration of coupons. In addition, a considerable empirical literature has studied the effectiveness of coupons (for a review, see Kumar & Swaminathan, 2005).

Coupon face value. Research on the face value of coupons has received considerable attention (Anderson & Song, 2004; Martín-Herrán & Sigué, 2015; Narasimhan, 1984; Raju, 1995). Raju (1995) provided an intensive review of the early theoretical models of sales promotions.

The seminal paper by Narasimhan (1984) establishes coupons as a mechanism to price discriminate between loyal and non-loyal consumers. Assuming a model with two groups of consumers, he models the optimal coupon value as a trade-off between the gain from non-loyal consumers or switchers, and losses due to redemption by loyal consumers. Anderson and Song (2004) extend the work by Narasimhan (1984), adding heterogeneity in redemption costs (where high-value consumers have higher redemption costs), and find that, depending on the level of the redemption cost and difference in consumers' valuations, a firm may either increase or even reduce the regular price when offering a coupon.

Martín-Herrán and Sigué (2015) study a setting where a single manufacturer may offer a coupon to consumers and/or a similar-valued trade deal to retailers (push vs. pull promotions). They consider on-pack coupons whereby consumers receive a coupon for a second purchase at a reduced price, which may influence demand in both the first and the second period. They derive equilibrium strategies, first determining the first-period retail price and next the manufacturer's optimal coupon face value and trade deal. Overall, the manufacturer prefers coupons to trade deals.

Our research on coupon face value is most closely related to the work by Anderson and Song (2004). We extend this research in several ways. First, we propose a dynamic model (rather than the previous static models) in which consumers can redeem coupons over time. Second, we incorporate forgetting and stochastic redemption costs (i.e., costs that may vary over time). Finally, and most important, different from all previous work, we simultaneously study optimal coupon duration and face value.

Coupon duration. Very limited research has considered the optimal coupon decision. Krishna and Zhang (1999) propose an analytical model with two competing brands offering coupons that have either a short (one-period) or long (two-period) redemption period. Consumers use either a coupon for brand A, for brand B, or for both brands, and consumers have greater preferences for either brand A or B or are indifferent. The authors find the redemption rate is higher for longer-duration coupons. This tends to benefit smaller-share brands that benefit more from increased redemption from consumers who prefer the competing brand, relative to the decrease in profits from their loyal consumers who would purchase at a regular price. Our research on coupon duration is most closely related to the work by Krishna and Zhang (1999). However, different from their research, we also consider different face values, consumer forgetting, and stochastic redemption costs.

Trump (2016) conducted two experiments to measure consumers' attitudes toward short- and long-duration coupons (one day vs. one month) and found significantly lower attitudes toward the company for shorter-duration coupons; however, this effect was attenuated for a high face value (50% discount vs. 10%). Trump attributes these results to reactance theory, as consumers react adversely to the restrictions imposed by the short-duration coupons.

However, none of these papers have considered the coupon-duration decision together with the optimal face value, which is the focus of the current research.

Empirical research on coupon-redemption patterns. Ward and Davis (1978) find cumulative coupon-redemption rates increase in a concave manner with respect to time, suggesting the majority of coupons are redeemed early on and redemption reduces over time. Inman and McAlister (1994) also observe a decay in coupon-redemption rates over time, but this decay is followed by a spike close to the expiration date of the coupon. The authors explain this spike using regret theory, which suggests that as a coupon's expiration date approaches, regret about missing the deadline continues to increase.

Shu and Gneezy (2010) document empirical findings pertaining to the procrastination inherent in consumers' redemption behavior. They build upon resource slack theory (Zauberman & Lynch, 2005), which suggests greater discounting of time investments relative to money investments. The difference in discounting is attributed to people's incorrect belief that they will have more free time (slack) in the future than they have in the present; thus, such over-prediction of slack is more pronounced for time than for money. Shu and Gneezy (2010) use temporal construal theory (Trope & Liberman, 2003) to examine how non-monetary costs and benefits are treated in the short and long run. Temporal construal theory suggests individuals tend to focus on the desirability of a task in the long run but switch to a focus on the feasibility of the task as it approaches fulfillment. All of these theories suggest the surplus the consumer enjoys from a cost-benefit perspective will appear larger when it is pushed out to the future. Based on conceptualizations that invoke myopic behavior in consumers, Shu and Gneezy (2010) hypothesize and find that redemption rates are *higher* when consumers are assigned to shorter redemption windows. Surprisingly, despite lower redemption, Shu and Gneezy show consumers exhibit *higher* satisfaction with longer redemption windows (p. 939). Soman (1998) shows an empirical relationship between the redemption costs and the time to redeem. Specifically, the perceived aversion to effort decreased when the redemption window increased from "within the weekend" to "within two weeks."

In summary, empirical research indicates the majority of coupons are redeemed early on and the rate of redemption reduces over time, potentially with a spike just before the expiration date. Several researchers have used behavioral theories to try to explain these redemption patterns, assuming myopic consumers (i.e., consumers' myopic [naïve] understanding of their own future behavior is what drives observed behavior). O'Donoghue and Rabin (1999) observed *naïve* and *sophisticated* consumers, with the former exhibiting myopic perceptions about their future behavior and the latter exhibiting correct perceptions about their future behavior. By contrast, this current research assumes rational forward-looking consumers who will anticipate future events and act accordingly.[2]

3. Model 1: coupon-design decision when price is exogenous

To avoid confounding explanations, we start with a monopoly setting where a single brand or retailer offers a coupon to

[2] For example, Inman and McAlister (1994) implicitly assume consumers are myopic in that they do not anticipate or respond to the regret until they experience it. By contrast, rational forward-looking consumers will anticipate the "regret" or "loss" due to their failure to redeem a coupon, and will redeem the coupon at an earlier point in time.

consumers, with varying coupon durations (e.g., Anderson & Song, 2004). Such a monopolistic setup also has practical applications when coupons are used to stimulate demand from their own low-value customers.

This setting is most applicable in centralized supply chains in which a central controller coordinates the manufacturer's and retailer's pricing and coupon strategies to maximize channel profit, or when manufacturers sell directly to consumers or retailers, while offering coupons to consumers. Both strategies are frequently used. For example, manufacturers of electronics, such as Sony and Bose, sell directly to consumers and offer them promotional deals. Also, many retailers like Staples use coupons targeted to consumers.

In the model extension, we investigate the joint optimization of coupon duration and face value for two competing sellers in a duopoly setup.

Model 1 is cast in a monopoly setting where one brand is offered to the market in each selling period within a multi-period selling frame. Similar to Narasimhan (1984), we assume two types of consumers exist: H-type (high-value or loyal consumers) and L-type (low-value or coupon-prone consumers) with valuations V_H and V_L ($< V_H$), correspondingly. Each consumer (H- or L-type) purchases at most one product in each period. The market size of H-type consumers is n_H and the market size of L-type consumers is n_L. We assume the price is fixed and the seller determines the optimal coupon design, including coupon face value and duration. In Model 2, we relax this assumption and allow price to vary when coupons are offered.

The seller's regular selling price may either be high or low: $p = V_H$ or $p = V_L$. Clearly, when the regular price is $p = V_L$, both H-type and L-type consumers buy the product and the profit is $(n_H + n_L)V_L$. If a coupon is offered, profit will be less than $(n_H + n_L)V_L$; thus, offering a coupon is not profitable under this scenario. Therefore, we focus on the case $p = V_H$ in Model 1.[3]

The optimal face value (X) and the duration or expiration date of the coupon need to be determined. The duration can be one, two, or three periods, indicating short-, medium-, and long-duration coupons. Further, all periods are ex-ante identical, although specific realizations of costs vary across periods (i.e., stochastic redemption costs). This assumption has the appealing property that period characteristics are not driving our findings. We next list the key assumptions associated with our 320 model.

3.1. Forgetting/Recall

Forgetting has a rich history in communication models in which consumers simply forget about messages (Keller, 1987; Tellis, 1998). Moreover, the empirically observed declining pattern of coupon redemption over time fits well with the notion of forgetting (Inman & McAlister, 1994; Ward & Davis, 1978). Although the act of forgetting needs little justification, the specific functional form assumed for forgetting requires further support. We assume recall will be exponentially decaying over time, consistent with the finding by Rubin and Wenzel (1996), who conducted a meta-analysis of 210 published data sets on retrospective recall and found the exponential function provides the best fit.

We posit that both H- and L-type consumers will decide whether to redeem a coupon immediately or postpone the redemption, trading off the redemption costs with the benefits (face value). Consumers who face high redemption costs may postpone redemption to a later period; however, some consumers who do so will forget to redeem the coupon. We assume the probability that the consumer will recall the coupon is proportional to the face value of the coupon (X) and equal to $Min(\alpha_i X,\ 1)$, ($i = H$ or L) for both types of consumers; conversely, the probability that the consumer will forget about a coupon is $1 - Min(\alpha_i X,\ 1)$, where α_i indicates the strength of recall.[4] This is mathematically identical to the exponential decay rate identified in previous research. We further assume that once consumers forget about the coupon, they will not recall the offer in future periods given that they receive many coupons, through multi-channels including emails, mobile Apps, and coupon pamphlets.[5] In addition, because all periods are identical, the probability of transitioning across any two periods with retention of the offer is $Min(\alpha_i X,\ 1)$.

3.2. Stochastic redemption costs

We next posit that redemption costs for a coupon may vary by period for a consumer. This is consistent with empirical observations that perceived time pressure is an important determinant of the extent of consumers' search for coupons (Vermeir & Van Kenhove, 2005). That is, whether consumers check to see if they have a coupon for a product before purchasing is contingent on how pressed they are for time during that particular shopping trip. In effect, time pressure exacerbates the costs of redeeming a coupon.

Further, we expect these costs to vary stochastically over time as consumers' situational factors vary from period to period (Chen et al., 2005; O'Donoghue & Rabin, 1999). For example, some periods may realize more demand at work or at home, leading to high redemption costs. Other periods have more slack time, and redemption costs are low. Also, the costs of redeeming a coupon (the effort to extract and organize the right coupon and take the coupon to the store) tend to be lower when consumers conduct a "planned" shopping trip, and are high for a "rushed" shopping trip (Vermeir & Van Kenhove, 2005). Following this discussion, we posit that redemption costs are stochastic in that they may be either low or high for both types of consumers (c_{il} or c_{ih}; $i = H$ or L), and $c_{Lh} \le c_{Hh}$, because L-type (coupon-prone) consumers have more experience using coupons. Without loss of generality, we set $c_{il} = 0$ for both H- and L-type consumers. Moreover, we assume that in each period, which are ex-ante identical, the probability of the redemption costs being low is β_i, ($i = H$ or L) and $0 \le \beta_i \le 1$; conversely, the probability of the redemption costs being high is $1 - \beta_i$. The nature of this specification for stochastic aspects of the utility function is similar to that employed by O'Donoghue and Rabin (1999) and Chen et al. (2005).

3.3. Information structure and timing

We assume all parameters, namely, V_i, X, α_i, β_i, c_{il}, and c_{ih}, are known to both types of consumers, but the outcome of the costs in each period is stochastic and is unknown beforehand. Moreover, at the beginning of every period, consumers become aware of the realization of their redemption costs for that period. Redemption costs in future periods are only known probabilistically. Consumers are issued a coupon in the first period, and we assume forgetting commences only with the second period. Again, the basic elements of this information structure and timing are similar to those assumed in the works of O'Donoghue and Rabin (1999) and Chen et al. (2005).

[3] Setting the regular price at $p = V_H$ when $n_H V_H > (n_H + n_L)V_L$ is optimal for the seller.

[4] We use $Min(\alpha_i X,\ 1)$ because consumers' recall rate cannot exceed 1, to avoid a recall rate of more than 100% for coupons with a large face value.

[5] This assumption is similar to that employed in previous research and is a useful simplification for the purpose of understanding the basic nature of redemption behavior (Inman & McAlister, 1994; Shu & Gneezy, 2010; Soman, 1998). This simplification does not qualitatively affect our key findings (see Online Appendix A).

Table 1
Redemption rates across periods for different coupon durations When $Max(V_H - V_i, c_{ih}) \le X < V_H - V_i + c_{ih}$.

	One-period coupon	Two-period coupon	Three-period coupon
Period 1	β_i	β_i	β_i
Period 2		$Min(\alpha_i X,\ 1)(1-\beta_i)\beta_i$	$Min(\alpha_i X,\ 1)(1-\beta_i)\beta_i$
Period 3			$Min(\alpha_i X,\ 1)^2(1-\beta_i)^2\beta_i$

We next analyze forward-looking consumers' utility-maximization decisions over the entire planning horizon, including decisions on whether to purchase the product and whether to redeem the coupon. As discussed above, we know that when offering a coupon is optimal, only L-type consumers who only purchase using a coupon make the decision to buy. H-type consumers purchase every period but decide whether to redeem the coupon immediately or wait. The decision to wait or not depends on the level of realized redemption costs (i.e., whether consumers are in a low- or a high-cost state).

3.3.1. Forward-looking consumers' coupon-redemption behavior

We first list the possible moves and utilities for both consumer types, given different redemption costs. The regular product price is $p = V_H$. When redemption costs are high for both types of consumers, c_{ih}, the consumers have three options, with corresponding utilities:

$$\begin{cases} a.\ \text{buy with using coupon}, & \Rightarrow U_{ia} = V_i - p + (X - c_{ih}) \\ & \quad = V_i - V_H + (X - c_{ih}) \\ b.\ \text{buy without using coupon}, & \Rightarrow U_{ib} = V_i - p = V_i - V_H \\ c.\ \text{no purchase}, & \Rightarrow U_{ic} = 0. \end{cases} \tag{1}$$

When redemption costs are low, c_{il} (recall $c_{il} = 0$), consumers have two options:

$$\begin{cases} d.\ \text{buy with using coupon}, & \Rightarrow U_{id} = V_i - p + X = V_i - V_H + X \\ e.\ \text{no purchase}, & \Rightarrow U_{ie} = 0 \end{cases}. \tag{2}$$

An important question is whether a consumer will redeem the coupon immediately, or postpone redemption until the next period. Because redemption is a tradeoff between the costs (redemption) and benefits (face value), we next investigate the consumer's coupon-redemption behavior, depending on whether the coupon's face value (X) is higher or lower than the redemption costs.

Ø If face value is no less than cost, $X \ge c_{ih}$, consumers will postpone coupon redemption when

A. $\boldsymbol{U_{ia}} < 0$ **and** $\boldsymbol{U_{id}} \ge 0$. $\Rightarrow Max(V_H - V_i, c_{ih}) \le X < V_H - V_i + c_{ih}$. For $U_{ia} < 0$ and $U_{id} \ge 0$, consumers will only redeem when costs are low, $c_{il} = 0$. The redemption rate in each period for either a one-, two-, or three-period coupon is shown in Table 1. We call these redemption rates a Mode-1 pattern. A Mode-1 pattern is one in which the redemption rate has a monotonic downward trend, such that the majority of coupons are redeemed early on and redemption reduces over time (Ward & Davis, 1978). We also define a Mode-0 pattern for the instance in which all consumers redeem their coupon immediately in the first period. Finally, we define a Mode-2 pattern, a redemption rate with a monotonic downward trend followed by a spike close to the expiration date of the coupon (Inman & McAlister, 1994).

Clearly, for a Mode-1 pattern, the total redemption rate of a two-period coupon exceeds that of a one- period coupon; moreover, the redemption for a three-period coupon exceeds that for a two-period coupon, because $\beta_i + Min(\alpha_i X,\ 1)(1-\beta_i)\beta_i > \beta_i$and $\beta_i + Min(\alpha_i X,\ 1)(1-\beta_i)\beta_i + Min(\alpha_i X,\ 1)^2(1-\beta_i)^2\beta_i > \beta_i + Min(\alpha_i X,\ 1)(1-\beta)\beta_i$. In the Mode-1 pattern, for both the two- and three-period coupon, a downward trend exists in the redemption rate, where $\beta_i > Min(\alpha_i X,\ 1)(1-\beta_i)\beta_i > Min(\alpha_i X,\ 1)^2(1-\beta_i)^2\beta_i$, because $Min(\alpha_i X,\ 1) \le 1$ and $\beta_i \le 1$. This results in our first proposition:

Proposition 1. *When* $Max(V_H - V_i, c_{ih}) \le X < V_H - V_i + c_{ih}$, *consumers will postpone coupon redemption when facing high costs* c_{ih} *and redeem when costs are low. The longer the duration of the coupon, the higher the overall redemption rate. Moreover, for both two- and three-period coupons, the redemption rate reduces over time.*

Proposition 1 demonstrates a relatively strong condition for consumers to postpone their coupon redemption, because when $X < V_H - V_i + c_{ih}$, they won't receive positive utility from redeeming the coupon if the realized redemption cost is high, c_{ih}. And the condition $X \ge V_H - V_i$ ensures they will redeem the coupon when the realized redemption cost is low. In case A in Proposition 1, consumers do not need to strategically evaluate and compare the gains from redeeming immediately or postponing. Next, in case B, we investigate whether those consumers may strategically postpone redemption when utility from redeeming the coupon is not negative, say, $X \ge V_H - V_i + c_{ih}$, when the realized redemption cost is high, c_{ih}.

B. $\boldsymbol{U_{ia}} \ge 0$ **and** $\boldsymbol{U_{id}} \ge 0$. $\Rightarrow X \ge V_H - V_i + c_{ih}$. Consumers will redeem a coupon when the realized redemption costs are high, because $U_{ia} \ge 0$. The decision of when to redeem a coupon is a tradeoff between the benefit from instant redemption and the payoff from waiting. We next solve the consumer's coupon-redemption decision through backward deduction for the three-period-coupon scenario.

For a three-period coupon, if the consumer arrives in the third (final) period with a coupon, she redeems the coupon irrespective of whether the realized redemption costs are high or low, because $U_{ia} \ge 0$ and the coupon will expire after this period. If the consumer arrives in the second period with a coupon, the redemption decision will depend on the redemption cost. If redemption costs are low, the consumer redeems now because she cannot benefit from waiting. However, if redemption costs are high, the consumer has to consider the probability that redemption costs will be low with probability β_i in the next period, but also the chance of forgetting, $1 - Min(\alpha_i X,\ 1)$. The consumer will postpone redemption as long as

$$\begin{aligned} V_i - V_H + (X - c_{ih}) \ < \ & Min(\alpha_i X,\ 1)\ [\beta_i\ (V_i - V_H + X) \\ & + (1 -\ \beta_i)(V_i - V_H + (X - c_{ih}))] \\ & \Rightarrow V_H > V_i + X - c_{ih} - \frac{Min(\alpha_i X,\ 1)\beta_i c_{ih}}{1 - Min(\alpha_i X,\ 1)}. \end{aligned}$$

Combining this with conditions $X \ge V_H - V_i + c_{ih}$ and $X \ge c_{ih}$, we get $X \ge V_H - V_i + c_{ih}$ and $\gamma_i(X) < V_H$.[6] The first condition, $X \ge V_H - V_i + c_{ih}$, is to ensure the coupon face value is large enough such that i-type consumers will redeem the coupon when the redemption costs are high c_{ih}. The second condition, $\gamma_i(X) < V_H$, ensures that not all consumers will redeem the coupon immediately. Otherwise, when $\gamma_i(X) \ge V_H$, i-type consumers will redeem immediately during the first period.

Finally, because $U_{ia} \ge 0$, i-type consumers will always redeem when the duration is one period. Hence, the redemption rates per period for one-, two-, and three-period coupons are shown in Table 2.

The redemption pattern shown above is a Mode-2 pattern. Different from the Mode-1 pattern, the redemption rates do not necessarily follow a downward pattern across all periods (i.e., coupon

[6] For the purpose of succinctness, here we define $\gamma_i(X) = V_i + X - c_{ih} - \frac{Min(\alpha_i X,\ 1)\beta_i c_{ih}}{1-Min(\alpha_i X,\ 1)}$.

Table 2
Redemption rates across periods for coupons with different durations when $X \geq V_H - V_i + c_{ih}$ and $\gamma_i(X) < V_H$.

	One-period coupon	Two-period coupon	Three-period coupon
Period 1	1	β_i	β_i
Period 2		$Min(\alpha_i X,\ 1)(1-\beta_i)$	$Min(\alpha_i X,\ 1)(1-\beta_i)\beta_i$
Period 3			$Min(\alpha_i X,\ 1)^2(1-\beta_i)^2$

Table 3
Summary of consumers' coupon-redemption patterns.

Face value (X) vs. redemption costs	Net benefit from coupon	Redemption pattern over time
$X \geq c_{ih}$	$V_H - V_i \leq X < V_H - V_i + c_{ih}$	Mode-1
	$X \geq V_H - V_i + c_{ih}$ and $\gamma_i(X) < V_H$	Mode-2
	$X \geq V_H - V_i + c_{ih}$ and $\gamma_i(X) \geq V_H$	Mode-0
$X < c_{ih}$		Mode-1

redemptions exhibit a monotonic downward trend over time, but with a potential spike in the redemption rate at the expiration period). We characterize this redemption pattern in Propositions 2a and 2b.

Proposition 2a. *When $X \geq V_H - V_i + c_{ih}$ and $\gamma_i(X) < V_H$, i-type consumers will postpone coupon redemption when facing high redemption costs c_{ih}. However, they will always redeem the coupon in the last period irrespective of whether the redemption costs are high or low. Specifically, when $Min(\alpha_i X,\ 1) > \frac{\beta_i}{1-\beta_i}$, the redemption rate will increase from Period 1 to Period 2 for a two-period coupon, whereas for a three-period coupon, the redemption rate first decreases from Period 1 to Period 2 and then increases (a spike) from Period 2 to Period 3. However, the total redemption rate always decreases with the length of the coupon duration.*

Proposition 2b. *When $X \geq V_H - V_i + c_{ih}$ and $\gamma_i(X) \geq V_H$, i-type consumers will redeem a coupon immediately in the first period even when redemption costs c_{ih} are high.*

The first condition $X \geq V_H - V_i + c_{ih}$ in Proposition 2a indicates consumers (i -type) can receive positive utility from redeeming a coupon when the realized redemption cost is high. Therefore, the only reason to strategically postpone redemption is when the gain from waiting is larger, which is ensured by condition $\gamma_i(X) < V_H$. As we discussed above, this condition $\gamma_i(X) < V_H$ is simultaneously determined by consumers' forgetting behavior and their stochastic redemption costs, and this condition varies according to different types of consumers. Because different types of consumers will have different redemption patterns, while facing the same coupon, our findings in Propositions 1, 2a, and 3 (when $X < c_{ih}$ shown below) provide a new angle to segment the market via coupons while considering both the coupon face value and the coupon duration.

Proposition 2a mirrors the fundamental notion of decay associated with redemption as well as a spike in redemption in Period 3 just before expiration. Whereas the former stems straightforwardly from forgetting and diminished redemption over time, the latter result provides new theoretical support for the empirical findings established in Inman and McAlister (1994). Note that $Min(\alpha_i X,\ 1) > \frac{\beta_i}{1-\beta_i}$ will only hold when the probability of low redemption costs is sufficiently low (i.e., the right-hand side monotonically decreases in β_i) and/or recall is high enough. Under those circumstances, more consumers will postpone their redemption, resulting in a spike in redemption in the last period.

Next, we characterize total redemption for one-, two-, and three-period coupons. Given the results in Proposition 2a, the overall redemption rates for one-, two-, and three-period coupons are b 1, $\beta_i + Min(\alpha_i X,\ 1)(1-\beta_i)$, and $\beta_i + Min(\alpha_i X,\ 1)\beta_i(1-\beta_i) + Min(\alpha_i X,\ 1)^2(1-\beta_i)^2$, correspondingly. Using simple algebra, we provide theoretical support for the counterintuitive notion that redemption rates are higher for shorter-duration coupons, consistent with results by Shu and Gneezy (2010).

Finally, when the coupon's face value is sufficiently high, $X \geq V_H - V_i + c_{ih}$ but $\gamma_i(X) \geq V_H$, i-type consumers do not benefit from waiting; instead, they will redeem the coupon immediately in the first period. We call this redemption pattern a Mode-0 pattern.

C. U_{id} < 0. Because $U_{ia} < U_{id} < 0$, consumers will not buy either with or without a coupon. Therefore, we exclude this case.

Ø If $X < c_{ih}$, consumers postpone redemption if redemption costs are c_{ih}.

When $U_{id} < 0$, they will not buy, even if costs are c_{il}. Therefore, we exclude this case and only considerwhen $U_{id} \geq 0$, which implies $X \geq V_H - V_i$. The redemption rate for these consumers in each period is identical to that exhibited in Table 1. Consequently, we have the following:

Proposition 3. *When $V_H - V_i \leq X < c_{ih}$, i-type consumers will postpone redemption when costs are high c_{ih} and only redeem when costs are low. Moreover, consumers' redemption rate for long-duration coupons follows a Mode-1 pattern, monotonically increasing with duration.*

We summarize consumers' redemption patterns stated in Proposition 1, 2, and 3 in Table 3, for two different ranges of coupon face values. Results show the different redemption patterns obtained over time, including monotonic downward trends and a potential spike just before the coupon expiration date.

As discussed above, the decision to redeem is a trade-off between costs and benefits, which is related to the expected redemption costs in the next period. Next, we want to examine if consumers can benefit from a long-duration coupon, that is, whether consumer surplus is maximized with a two-period or a three-period coupon. Consumer surplus is defined as the total benefit obtained by all consumers from redeeming coupons. On the one hand, the three-period coupon allows for a greater opportunity to wait, and consumers benefit from redeeming at their most opportune period. On the other hand, under certain circumstances, the total redemption rate can be higher for a two-period coupon (see Proposition 2a). Considering these counter forces, we formally state the impact of redemption length on consumer surplus in Proposition 4.

Proposition 4. *Consumer surplus is always higher for longer-duration coupons.*

Proof of Proposition 4: The total redemption rate, for a Mode-1 pattern, increases with coupon duration; hence, consumer surplus increases with duration. However, for a Mode-2 pattern, two-period coupons' overall redemption rate may be higher when $X \geq c_{ih}$. Specifically, for a Mode-2 pattern, all consumers redeem a one-period coupon; for a two-period coupon, the rate of redemption is $\beta_i + Min(\alpha_i X,\ 1)(1-\beta_i)$; and for a three-period coupon, the rate of redemption is $\beta_i + Min(\alpha_i X,\ 1)\beta_i(1-\beta_i) + Min(\alpha_i X,\ 1)^2(1-\beta_i)^2$. The associated surplus for a one-, two-, and three-period coupon is $\beta_i(V_i - V_H + X) + (1-\beta_i)(V_i - V_H + X - c_{ih})$; $\beta_i[V_i - V_H + X] + Min(\alpha_i X,\ 1)(1-\beta_i)[\beta_i(V_i - V_H + X) + (1-\beta_i)(V_i - V_H + X - c_{ih})]$; and $\beta_i[V_i - V_H + X] + Min(\alpha_i X,\ 1)\beta_i(1-\beta_i)[V_i - V_H + X] + Min(\alpha_i X,\ 1)^2(1-\beta_i)^2[\beta_i(V_i - V_H + X) + (1-\beta_i)(V_i - V_H + X - c_{ih})]$, respectively. Simplifying and comparing, we find that, conditional on $X \geq V_H - V_i + c_{ih}$ and $V_H > \gamma_i(X)$ shown in Table 3, the surplus for a three-period coupon exceeds that of a two-period coupon, and the surplus for a two-period coupon exceeds that of a one-period coupon. Hence, the longer the coupon duration, the higher the consumer surplus. This finding suggests consumers benefit from longer redemption windows, which provides them with more flexibility in scheduling the best time to redeem, even though redemption rates are lower. Hence, Proposition 4 provides a rationale for why consumers may be more satisfied with longer-duration coupons (Shu & Gneezy, 2010).

3.3.2. Optimal coupon face value and duration

We analyze the optimal coupon design with respect to face value and duration given the consumers' strategic coupon-redemption behavior stated in the previous section. The seller can issue a coupon with either, a one-, two-, or three-period duration.

Because H-type consumers purchase one product in each period at $p = V_H$, and L-type consumers only buy once using a coupon, the seller gains from coupon redemptions by L-type consumers and loses from redemption by H-type consumers. The seller's objective is to jointly determine the face value and coupon duration, such that L-type consumers will use a coupon while considering the gains and losses from the different types of consumers. This is the "net benefit" a seller tries to maximize. We next examine the optimal strategies with and without a coupon. Clearly, when a coupon is not offered, the net benefit of using a coupon is zero.

Next, we investigate the coupon strategy. A coupon is distributed to the market with a duration of either one, two, or three periods, corresponding to a short-, medium-, or long-duration coupon. The driving force behind using a coupon is that H- and L-type consumers have different redemption behaviors. The seller's net benefit can be maximized by choosing the coupon's face value X and duration, such that L-type consumers will purchase with a coupon, while preventing H-type consumers from using the coupon. We first demonstrate some lemmas related to offering coupons:

Lemma 1. *The face value of a coupon should not be lower than $V_H - V_L$, $X \geq V_H - V_L$.*

The logic of Lemma 1 is straightforward. A necessary condition for coupons to be profitable is that L-type consumers should buy with a coupon when redemption costs are low ($c_{Ll} = 0$) or not buy at all, indicating $U_{Ld} = V_L - p + X = V_L - V_H + X \geq 0$, which implies $X \geq V_H - V_L$.

Lemma 2. *For a one-period coupon, the face value should be lower than the redemption costs of H-type consumers, $X < c_{Hh}$, to prevent them from using a coupon at c_{Hh}. Consequently, the redemption rate of H-type consumers is β_H for one-period coupons.*

Lemma 2 is understood as follows. For a one-period coupon, if H-type consumers will buy the product with a coupon when facing c_{Hh}, they will also buy when facing c_{Hl}. Therefore, H-type consumers will always buy with a coupon. Lemma 2 ensures H-type consumers will not always buy using a coupon. Because $X \geq V_H - V_L$ as stated in Lemma 1, we get $V_H - X \leq V_L$, which indicates that for each unit sold to H-type consumers, profits are not more than V_L. Clearly, this coupon strategy is worse than selling to both types at $p = V_L$ without offering a coupon. Consequently, for a one-period coupon to be optimal, H-type consumers will only use a coupon when facing c_{Hl}, and their redemption rate is β_H.

Lemma 3. *For a two- or three-period coupon, postponing redemption when facing high redemption costs, c_{Hh}, is optimal for H-type consumers.*

Similar to the proof of Lemma 2, for two- and three-period coupons, if H-type consumers do not postpone coupon redemption when facing c_{Hh}, they will always redeem the coupon in the first period. Because $X \geq V_H - V_L$, as shown in Lemma 1, the profit per unit sold to H- and L-type consumers who redeem a coupon is no more than V_L, and the total profit from them is no higher than $(n_H + n_L)V_L$. Again, because $n_H V_H \geq (n_H + n_L)V_L$, this strategy is not better than selling to H-type consumers at $p = V_H$ without offering a coupon.

For net-benefit maximization, the coupon face value and duration need to be simultaneously determined, and thus we have Proposition 5:

Proposition 5. *Coupon duration and face value jointly influence the ability to use coupons to price discriminate between H- and L-TYPE consumers. Specifically,*

Proposition 5a. *Using a one-period coupon is never optimal when $V_H - V_L \geq c_{Hh}$. Moreover, the coupon's face value can never exceed the high redemption costs of the H-type, $X < c_{Hh}$.*

Proposition 5b. *The face value of longer-duration coupons can be higher than c_{Hh}. Specifically, longer-duration coupons may be optimal when $X \geq Max(V_H - V_L, c_{Hh})$ and $\gamma_H(X) < V_H$ or $V_H - V_L \leq X < c_{Hh}$.*

To prove Proposition 5, we calculate the optimal design for a one-period coupon and the corresponding net benefit (details are provided in Appendix A). As shown in Table 4, we find a one-period coupon is not an optimal strategy when $V_H - V_L \geq c_{Hh}$, because when the valuation gap between H- and L-type consumers is larger than or equal to c_{Hh}, the two conditions $X \geq V_H - V_L$ and $X < c_{Hh}$, as stated in Lemmas 1 and 2, cannot coexist. Consequently, this provides proof for Proposition 5a.

Optimality of two- and three-period coupons. Lemma 3 shows that for long-duration coupons, we need to make sure H-type consumers postpone redemption at c_{Hh}. As demonstrated in Propositions 1 and 2a, consumers will postpone coupon redemption when facing high costs c_{ih} when $Max(V_H - V_i, c_{ih}) \leq X < V_H - V_i + c_{ih}$ and $X \geq V_H - V_i + c_{ih}$. Therefore, setting $V_i = V_H$, H-type consumers will postpone redemption when $c_{Hh} \leq X < c_{Hh}$ or when $X \geq c_{Hh}$ and $\gamma_H(X) < V_H$.[7] Clearly, the case $c_{Hh} \leq X < c_{Hh}$ cannot exist. Moreover, Proposition 3 demonstrates that when the coupon's face value is low, $X < c_{Hh}$, H-type consumers will always postpone redemption at c_{Hh}. Therefore, combining the two conditions—$X \geq c_{Hh}$ and $\gamma_H(X) < V_H$, and $X < c_{Hh}$ with $X \geq V_H - V_L$ (from Lemma 1)—we obtain Proposition 5b.

Analyses of the coupon strategy for long-duration coupons focuses on the two constraints stated in Proposition 5b (details are provided in Appendix B). Based on this analysis, we conclude that both coupon face value and duration, given stochastic redemption costs, can significantly moderate L-type and H-type consumers' redemption behavior, and as such have an impact on the ability to use coupons to price discriminate. Furthermore, the conditions and strategies for using long-duration coupons are listed in Table 4 (corresponding net benefits are provided in Appendix C).

Table 4 indicates both L- and H-type consumers' coupon-redemption modes (Mode-0, Mode-1, or Mode-2) vary for different coupon face values, and as a result, the optimal coupon duration also varies. Based on these findings, the maximization of net benefits is as follows. First, the optimal coupon face value and corresponding net benefit of a one-period coupon are determined as shown in Table 4 and Appendix A. Next, the optimal coupon face value for two- or three-period coupons is determined. For each constraint on the coupon face value (constraints on X in Appendix C), the coupon duration is determined based on L- and H-type consumers' redemption patterns, which can either be two periods or three periods or both. Next, we maximize

[7] $\gamma_H(X) = V_H + X - c_{Hh} - \frac{Min(\alpha_H X,\ 1)\beta_H c_{Hh}}{1 - Min(\alpha_H X,\ 1)}$ and $\gamma_L(X) = V_L + X - c_{Lh} - \frac{Min(\alpha_L X,\ 1)\beta_L c_{Lh}}{1 - Min(\alpha_L X,\ 1)}$ in the rest of the analyses.

Table 4
The optimal design of one-period coupons and *N*-period coupons.

Condition	One-period: Optimal coupon face value (X^*)	*N*-period: Strategies[a]	Redemption pattern[b]	Duration (in periods)
Small valuation gap : $V_H - V_L \le c_{Hh} - c_{Lh}$	$X^* = V_H - V_L + c_{Lh}$ $X^* = V_H - V_L$	*N4*	L: Mode-1 H: Mode-1	2 or 3
Medium valuation gap: $c_{Hh} - c_{Lh} < V_H - V_L \le c_{Hh}$	$X^* = V_H - V_L$	*N1*	L: Mode-1 H: Mode-2	3
		N2	L: Mode-2 H: Mode-2	2 or 3
		N3	L: Mode-0 H: Mode-2	3
		N4	L: Mode-1 H: Mode-1	2 or 3
Large valuation gap: $V_H - V_L > c_{Hh}$	N/A	*N1*	L: Mode-1 H: Mode-2	3
		N2	L: Mode-2 H: Mode-2	2 or 3
		N3	L: Mode-0 H: Mode-2	3

[a] Different strategies related to specific constraints specified in Appendix C, with details about the net benefit and corresponding optimal face value. Strategies *N2* and *N3* with condition $V_H - V_L \le c_{Hh} - c_{Lh}$ (small valuation gap) may be an optimal *N*-period strategy. However, compared to a one-period coupon strategy with $X^* = V_H - V_L + c_{Lh}$, both *N2* and *N3* have a higher face value, higher redemption from H-type, and lower redemption from L-type. Hence, we only consider *N4* as the *N*-period strategy for this condition. In addition, when $V_H - V_L > c_{Hh}$ (large valuation gap), strategy *N4* will not emerge, because it needs $V_H - V_L < c_{Hh}$.
[b] Coupon-redemption pattern for L-type (L) and H-type (H).

the corresponding net benefits and determine the local optimums for coupon face value under each constraint (optimal face value X^* in Appendix C). Finally, the global optimal face value and coupon duration are determined by comparing the local optimums across different duration levels.

Comparing profitability of one-period versus *longer-period coupons.* As stated in Proposition 5a, the coupon's face value should never exceed the high redemption costs of the H-type, $X < c_{Hh}$, and because of this constraint, a one-period coupon is never optimal when $V_H - V_L \ge c_{Hh}$. However, a longer-duration coupon may result in H-type consumers postponing redemption when the coupon face value is higher than c_{Hh} (see Proposition 1 and 2a). As such, a long-duration coupon might relax the constraint $X < c_{Hh}$ from a one-period coupon, because not all H-type consumers will redeem. Therefore, offering longer-duration coupons becomes feasible.

Another advantage of longer-duration coupons is that more L-type consumers may purchase. With a one-period coupon and $X^* = V_H - V_L$ (see Appendix A), only a portion of the L-type consumers, β_L, buy the product. However, according to Table 1 (setting type $i = L$), a longer-duration coupon will increase the redemption from L-type consumers beyond the portion β_L, resulting in an increase in net benefit. However, the cost of using long-duration coupons is that more H-type consumers may redeem a coupon (because the redemption rate is β_H for one-period coupons, see Lemma 2 and Table 4).

We formally compare the net benefits of long-duration coupons (in Appendix C) with the net benefit of one-period coupons (in Appendix A). Specifically, we determine the conditions under which, for recall of H-type consumers, two-period and three-period coupons are more profitable than one-period coupons. (We select recall of H-types because this factor is key in determining the optimal duration). Analyses provided in Online Appendix B show both two-period and three-period coupons are more profitable than one-period coupons when recall by H-type consumers is sufficiently small as stated in Proposition 6. This result holds for the redemption modes in Table 4. Basically, longer-duration coupons benefit more from increased redemption by L-types than they are hurt from the loss due to redemption by H-types, as long as recall is sufficiently small. Moreover, for a Mode-2 pattern (longer duration and lower redemption) for H-types or a Mode-1 pattern (longer duration and higher redemption) for L-type consumers, a three-period coupon is more profitable than a two-period coupon, and vice versa. Hence, the seller needs to trade off these potential gains and losses to determine the optimal coupon duration.

Proposition 6. *When prices are endogenous, for long-duration coupons to be optimal, the level of recall by H-type consumers, α_H, should be sufficiently low.*

Redemption patterns. Findings in Table 4 indicate H- and L-type consumers may exhibit different redemption patterns for coupons with varying durations, even for the same face value. Both types can experience Mode-1 or Mode-2 patterns, or one type may experience a Mode-1 (or Mode 0) and the other a Mode-2 pattern, depending on the coupon face value. The overall coupon-redemption pattern across different selling periods is a combination of the redemption patterns of each type of consumer. Recall the Mode-1 redemption pattern exhibits a monotonic downward trend across periods, and the Mode-2 redemption pattern exhibits a spike close to the expiration of the redemption period when $Min(\alpha_i X,\ 1) > \frac{\beta_i}{1-\beta_i}$, and a monotonic downward trend when $Min(\alpha_i X,\ 1) \le \frac{\beta_i}{1-\beta_i}$ (see Propositions 1 and 2b). The aggregate redemption curves showing the combined consumers' redemption patterns are summarized in Table 5.

Numerical analysis. In addition, we conducted numerical analysis examining the optimal coupon strategy for different levels of valuations for both consumer types. We focus on redemption patterns and observe monotonic decreasing redemption patterns and patterns with a spike at redemption in the final period for both two- and three-period coupons. Detailed results and the procedure used are discussed in Online Appendix C.

4. Model 2: Coupon-design decision when price is endogenous

In Model 2, we relax the assumption that price is fixed and allow it to vary during the time the coupon is offered. Here, in

Table 5
Summary of coupon-redemption patterns across different consumer types.

L-type redemption pattern	H-type redemption pattern	Overall redemption pattern (consumer types combined)
Mode-1	Mode-1	Monotonic downward trend
Mode-1 or Mode-0	Mode-2 but $Min(\alpha_H X,\ 1) \le \frac{\beta_H}{1-\beta_H}$	
Mode-2 but $Min(\alpha_L X,\ 1) \le \frac{\beta_L}{1-\beta_L}$	Mode-1	
Mode-2 but $Min(\alpha_L X,\ 1) \le \frac{\beta_L}{1-\beta_L}$	Mode-2 but $Min(\alpha_H X,\ 1) \le \frac{\beta_H}{1-\beta_H}$	
Mode-2 but $Min(\alpha_L X,\ 1) > \frac{\beta_L}{1-\beta_L}$	Mode-2 but $Min(\alpha_H X,\ 1) > \frac{\beta_H}{1-\beta_H}$	A spike in the redemption rate in the last period
Mode-0	Mode-2 but $Min(\alpha_H X,\ 1) > \frac{\beta_H}{1-\beta_H}$	
Mode-1	Mode-2 but $Min(\alpha_H X,\ 1) > \frac{\beta_H}{1-\beta_H}$	Undetermined
Mode-2 but $Min(\alpha_L X,\ 1) > \frac{\beta_L}{1-\beta_L}$	Mode-1	
Mode-2 but $Min(\alpha_L X,\ 1) > \frac{\beta_L}{1-\beta_L}$	Mode-2 but $Min(\alpha_H X,\ 1) \le \frac{\beta_H}{1-\beta_H}$	
Mode-2 but $Min(\alpha_H X,\ 1) \le \frac{\beta_H}{1-\beta_H}$	Mode-2 but $Min(\alpha_L X,\ 1) > \frac{\beta_L}{1-\beta_L}$	

Table 6
Summary of consumers' coupon-redemption patterns when p_c is endogenous.

Face value vs. redemption costs	Conditions on p_c	Redemption pattern over time
$X \ge c_{ih}$	$V_i + (X - c_{ih}) < p_c \le V_i + X$	Mode-1
	$V_i + X - c_{ih} - \frac{\alpha_i \beta_i c_{ih}}{1-\alpha_i} < p_c \le V_i + (X - c_{ih})$	Mode-2
	$p_c \le V_i + X - c_{ih} - \frac{\alpha_i \beta_i c_{ih}}{1-\alpha_i}$	Mode-0
$X < c_{ih}$		Mode-1

addition to the coupon value X and duration, the product's price during the coupon period p_c is also a decision variable. As mentioned above, in this model, the consumers' forgetting is assumed to be constant and equal to α_i, ($i = H$ or L) for both types of consumers. All other parameters are the same as in Model 1.

4.1. Forward-looking consumers' coupon-redemption behavior

We first list the possible moves for H- and L-type consumers with a coupon, and the corresponding utilities for high and low redemption costs. Following analyses similar to those in Model I, we find consumers' redemption patterns (results are summarized in Table 6) are consistent with results for Model 1 (Proposition 1, 2, 3), except that the results vary depending on conditions on p_c.

4.2. Optimal coupon face value and duration

We maximize the seller's profits by jointly determining face value and coupon duration given the above redemption behaviors. Here, we use profits rather than "net benefits," because coupon redemption and product price both affect the seller's profits. The model setup and parameters are identical to those for Model 1. Here, different from the exogenous-price case (Model 1), because the seller can vary the product price during the coupon period, offering a coupon when the regular product price is V_L might be optimal. Therefore, we first examine the benchmark strategies without a coupon:

Benchmark 1: Low-price strategy, $p_B = V_L$. Both H- and L-type consumers buy the product, and the seller's profits in each selling period are

$$\pi_{B1} = (n_H + n_L)V_L. \tag{3}$$

Benchmark 2: High-price strategy, $p_B = V_H$. Only H-type consumers buy the product, and the seller's profits in each period are

$$\pi_{B2} = n_H V_H. \tag{4}$$

Again, when a coupon is offered, the seller maximizes profits by encouraging sufficient L-type consumers to buy with a coupon, while preventing as many H-type consumers as possible from using the coupon. The optimal designs of one-period and N-period ($N=2$, 3) coupons are listed in Tables 7 and 8, respectively, with their profits and the valid conditions (details of the analyses are in Online Appendix D).

Results show that compared to benchmark 1, with price $p_B = V_L$, offering a coupon increases the price. However, compared to benchmark 2, with price $p_B = V_H$, offering a coupon sometimes results in a reduction in price: when $V_H - V_L > c_{Hh}$ and $V_H - V_L > c_{Hh} - c_{Lh}$ for one-period coupons and $V_H - V_L > c_{Hh}$ for N-period coupons. The reason is that when the valuation gap between H- and L-type consumers is large, the price targeted to H-types is relatively high for L-types, and a high face value is needed for purchase. However, if the coupon face value is higher than the redemption cost of H-type consumers, the seller needs to lower the price and select a face value lower than the redemption cost of H-type (this result is also consistent with results by Anderson & Song, 2004). For example, the seller needs to reduce the price if a one-period coupon is used (see Table 7). However, results in Table 8 (e.g., when $c_{Hh} < V_H - V_L < c_{Hh} + \frac{\alpha_H \beta_H c_{Hh}}{1-\alpha_H}$) show reducing the price for all durations is not necessary (even when the valuation gap is larger than the redemption cost of H-type consumers).

Overall, we find that selecting the right coupon duration can mitigate H-type consumers' redemption, while getting L-type consumers to redeem, without the need to lower the regular price.

Table 7
The optimal design, price, and profit for a one-period coupon.

One-period coupon		
Conditions	Optimal design: Face value and price	Profit
Small valuation gap:	$X^* = V_H - V_L,\ p_c^* = V_H$	$\beta_H n_H V_L + (1-\beta_H) n_H V_H + \beta_L n_L V_L$
$V_H - V_L \le c_{Hh} - c_{Lh}$	$X^* = V_H - V_L + c_{Lh},\ p_c^* = V_H$	$\beta_H n_H (V_L - c_{Lh}) + (1-\beta_H) n_H V_H + n_L (V_L - c_{Lh})$
Medium valuation gap:	$X^* = V_H - V_L,\ p_c^* = V_H$	$\beta_H n_H V_L + (1-\beta_H) n_H V_H + \beta_L n_L V_L$
$c_{Hh} - c_{Lh} < V_H - V_L \le c_{Hh}$	$X^* = c_{Hh},\ p_c^* = V_L + (c_{Hh} - c_{Lh})$	$\beta_H n_H (V_L - c_{Lh}) + (1-\beta_H) n_H [V_L + (c_{Hh} - c_{Lh})] + n_L (V_L - c_{Lh})$
Large valuation gap:	$X^* = c_{Hh},\ p_c^* = V_L + c_{Hh}$	$\beta_H n_H V_L + (1-\beta_H) n_H (V_L + c_{Hh}) + \beta_L n_L V_L$
$V_H - V_L > c_{Hh}$	$X^* = c_{Hh},\ p_c^* = V_L + (c_{Hh} - c_{Lh})$	$\beta_H n_H (V_L - c_{Lh}) + (1-\beta_H) n_H [V_L + (c_{Hh} - c_{Lh})] + n_L (V_L - c_{Lh})$

Table 8
The optimal design and price for an N-period coupon.

N-period coupon				
Conditions	Sub-conditions	Optimal design: Face value and price	Redemption pattern	Duration
Small valuation gap: $V_H - V_L < c_{Hh} - c_{Lh}$		$X^* = V_H - V_L$, $p_c^* = V_H$	L: Mode-1 H: Mode-1	2 or 3
Medium valuation gap: $c_{Hh} - c_{Lh} \le V_H - V_L < c_{Hh}$		$X^* = V_H - V_L$, $p_c^* = V_H$	L: Mode-1 H: Mode-1	2 or 3
	$V_H - V_L < c_{Hh} - c_{Lh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H}$	$X^* = V_H - V_L + c_{Lh}$, $p_c^* = V_H$	L: Mode-2 H: Mode-2	2 or 3
	$V_H - V_L < c_{Hh} - c_{Lh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H} - \frac{\alpha_L\beta_L c_{Lh}}{1-\alpha_L}$	$X^* = V_H - V_L + c_{Lh} + \frac{\alpha_L\beta_L c_{Lh}}{1-\alpha_L}$ $p_c^* = V_H$	L: Mode-0 H: Mode-2	3
Large valuation gap: $V_H - V_L > c_{Hh}$		$X^* = c_{Hh}$, $p_c^* = V_L + c_{Hh}$	L: Mode-1 H: Mode-1	2 or 3
	$V_H - V_L < c_{Hh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H}$	$X^* = V_H - V_L$, $p_c^* = V_H$	L: Mode-1 H: Mode-2	3
	$V_H - V_L < c_{Hh} - c_{Lh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H}$	$X^* = V_H - V_L + c_{Lh}$, $p_c^* = V_H$	L: Mode-2 H: Mode-2	2 or 3
	$V_H - V_L < c_{Hh} - c_{Lh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H} - \frac{\alpha_L\beta_L c_{Lh}}{1-\alpha_L}$	$X^* = V_H - V_L + c_{Lh} + \frac{\alpha_L\beta_L c_{Lh}}{1-\alpha_L}$ $p_c^* = V_H$	L: Mode-0 H: Mode-2	3

Moreover, accounting for consumers' forgetting and stochastic redemption costs, we offer an alternative way to price discriminate based on coupon face value and duration, without the need to reduce price. For certain regions where N-period coupons are optimal ($c_{Hh} < V_H - V_L < c_{Hh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H}$, $c_{Hh} - c_{Lh} < V_H - V_L < c_{Hh} - c_{Lh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H}$ and $c_{Hh} - c_{Lh} < V_H - V_L < c_{Hh} - c_{Lh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H} - \frac{\alpha_L\beta_L c_{Lh}}{1-\alpha_L}$), the seller can set the price as high as V_H for long-duration coupons, but not for one-period coupons. The reason is that this high price is combined with a high coupon face value ($X > c_{Hh}$) to motivate L-type consumers to purchase, and H-type consumers will redeem coupons irrespective of the redemption cost if the coupon duration is one period. By offering a longer-duration coupon, fewer H-types will redeem, due to forgetting. Given that H-types have different forgetting and redemption costs than L-type consumers, coupon duration facilitates a coupon's ability to price discriminate. Summarizing the above, we get the following:

Proposition 7. *When prices are endogenous, and for conditions* $c_{Hh} < V_H - V_L < c_{Hh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H}$, $c_{Hh} - c_{Lh} < V_H - V_L < c_{Hh} - c_{Lh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H}$ *and* $c_{Hh} - c_{Lh} < V_H - V_L < c_{Hh} - c_{Lh} + \frac{\alpha_H\beta_H c_{Hh}}{1-\alpha_H} - \frac{\alpha_L\beta_L c_{Lh}}{1-\alpha_L}$, *a seller can offer long-duration coupons, with a high price* V_H *and a coupon face value higher than* c_{Hh}, *which is not feasible for a one-period coupon.*

5. Model extension: duopoly case

In this section, we extend our model including competition between coupon promotions (Demirag, Keskinocak & Swann, 2011; Krishna & Zhang, 1999). We assume the presence of two competing sellers (S1 and S2) who sell substitute brands over a three-period time span. Each seller has a loyal segment with H-type consumers and market size n_{H1} and n_{H2}.

Similar to the setup in Model 1, H-type consumers' valuation is V_H and each seller's regular selling price is $p = V_H$. Each seller issues one coupon, with a certain face value and duration (one, two, or three periods), to compete for the L-type consumers with valuation V_L and market size n_L. Again, forgetting (to use a coupon) is proportional to the face value of the coupon (X) and is equal to $Min(\alpha_i X,\ 1)$, ($i = H$ or L) for both types of consumers. Stochastic redemption costs are the same as in the monopoly model except that we set $c_{Lh} = c_{Ll} = 0$ for tractability. This simplifying assumption results in immediate redemption by L-types, except when they have two coupons (one for each brand), then they will first redeem the coupon with the higher face value.

First, we consider each seller's optimal monopolistic strategy. According to results in Table 4 and Appendix A, when $c_{Lh} = c_{Ll} = 0$, the optimal design for a one-period coupon without competition is $X^* = V_H - V_L$, with net benefit $NB_1^*(n_{Hj}) = n_L V_L - \beta_H n_{Hj}(V_H - V_L)$, (subscript 1 denotes the coupon duration; $j = 1$ or 2). This strategy is optimal when $V_H - V_L < c_{Hh}$.

Next, when $c_{Lh} = 0$, we have $\gamma_L(X) = V_L + X$. Because Lemma 1 needs $X \ge V_H - V_L$ for a coupon to be offered, a long-duration coupon may be the monopolistic optimum when $V_H \le \gamma_L(X)$. Consequently, according to the results in Appendix C, we show the long-duration strategies $N1$, $N2$, and $N4$ are not possible. Only the three-period coupon strategy $N3$ can be an optimal monopolistic strategy, with net benefit $N3_3(n_{Hj}) = n_L(V_H - X) - [\beta_H + Min(\alpha_H X,\ 1)(1-\beta_H)\beta_H + Min(\alpha_H X,\ 1)^2(1-\beta_H)^2]n_{Hj}X$, ($j = 1$ or 2). Because $N3_3(n_{Hj})$ decreases with X, the maximized net benefit $N3_3^*(n_{Hj})$ emerges when X is at the lower bound of the constraints, $X \ge Max(V_H - V_L, c_{Hh})$ and $\gamma_H(X) < V_H \le \gamma_L(X)$.

Hence, we conclude each seller's optimal monopolistic strategy can either be a one-period or a three-period coupon. Three combinations are possible: Both sellers' (S1 and S2) optimal strategy is a one-period coupon; one seller's optimal monopolistic strategy is a one-period coupon, and the other's optimal monopolistic strategy is a three-period coupon; or both S1 and S2's optimal strategy is to offer a three-period coupon. We next analyze the competing strategies of the two sellers with respect to these three scenarios.

Scenario 1: Both S1 and S2's optimal monopolistic strategy is a one-period coupon.

In this case, both sellers can avoid competition, by each offering their coupons in different periods. Because L-type consumers will redeem their coupon immediately, each seller can pick one separate period to launch his/her coupon. Clearly, they will be better off by alternating their coupon promotions. Consequently, they may signal the timing of their coupon promotions to avoid head-on competition, and each of them receives the monopolistic net benefit of the optimal one-period coupon.

Scenario 2: One seller's optimal monopolistic strategy is a one-period coupon, and the other seller's optimal monopolistic strategy is a three-period coupon.

Under this scenario, similar to the logic in scenario 1, both sellers can avoid head-on competition and receive their optimal monopolistic net benefit. The seller whose optimal monopolistic strategy is a three-period coupon will offer it in the first period, whereas the other seller, whose optimal monopolistic strategy is a one-period coupon, will offer it either in the second or third period.

Scenario 3: Both S1 and S2's optimal monopolistic strategy is a three-period coupon.

Head-on competition may not be avoided, because both sellers want to offer their coupons in the first period, although they can avoid competition by offering a suboptimal one-period coupon.[8] Therefore, we have four sets of strategies for two sellers: Both sellers choose to compete; one seller chooses to compete and the other one chooses to avoid competition (two combinations); and both sellers choose to avoid competition. Next, we derive the sellers' payoffs for these four scenarios.

5.1. Both sellers choose to compete

Here, both sellers will compete head-on for the L-type consumers. L-type consumers' utility from redeeming a coupon (by each seller) equals $V_L + X - V_H$. A consumer will first redeem the coupon with the higher face value, and the seller with the lower value will suffer the loss from forgetting, because only $Min(\alpha_L X,\ 1)$ L-type consumers will redeem in the second period, resulting in the following net benefit:

$$N3_{3F}\left(n_{Hj}\right) = Min(\alpha_L X,\ 1)n_L(V_H - X) - [\beta_H + Min(\alpha_H X,\ 1)(1-\beta_H)\beta_H + Min(\alpha_H X,\ 1)^2(1-\beta_H)^2]n_{Hj}X.$$

The subscript F in $N3_{3F}(n_{Hj})$ indicates this is the net benefit for the seller who fails in competing for the L-type consumers' in the first period. Because $N3_{3F}(n_{Hj})$ decreases with X, the optimal X^* is the corner solution at the lower bound of the constraints $X \geq Max(V_H - V_L, c_{Hh})$ and $\gamma_H(X) < V_H \leq \gamma_L(X)$ for the optimal three-period coupon, for which the seller's maximized net benefit is $N3^*_{3F}(n_{H2})$.

Both sellers will compete in the first period for redemption from L-type consumers by increasing the coupon face value from the monopolistic optimum X^*. We assume $n_{H1} < n_{H2}$, which indicates the share of loyal consumers for S1 is lower than that for S2. Therefore, S2 is more sensitive to a higher coupon face value than S1 because S2's opportunity loss from H-type consumers' redemption is higher. Consequently, we conclude that a critical value, $\hat{X} > X^*$, exists above which S2 will stop competing because his/her payoff with this coupon will be worse than $N3^*_{3F}(n_{H2})$ even though he/she can win the competition in the first period.

By setting the coupon face value at $\hat{X}$, S1 deters S2 from competing in the first period.

However, the net benefit for S1 is also lower than the monopolistic optimum, because the coupon face value $\hat{X}$ is not at the lower bound of the constraints $X \geq Max(V_H - V_L, c_{Hh})$ and $\gamma_H(X) < V_H \leq \gamma_L(X)$. With this coupon face value, the net benefit for S1 becomes

$$N3^*_{3W}(n_{H1}) = n_L\left(V_H - \hat{X}\right) - [\beta_H + Min\left(\alpha_H \hat{X},\ 1\right)(1-\beta_H)\beta_H + Min\left(\alpha_H \hat{X},\ 1\right)^2(1-\beta_H)^2]n_{H1}\hat{X}.$$

[8] Another suboptimal strategy is to launch one three-period coupon in the second period. Then, L-type consumers obtain two coupons successively from two sellers in the first and second period and will redeem each immediately. However, H-types will have only two periods to redeem the second coupon, with redemption rate $\beta_H + Min(\alpha_H X,\ 1)(1-\beta_H)$ because $X \geq Max(V_H - V_L, c_{Hh})$ for the three-period coupon. Clearly, launching a three-period coupon in the second period is suboptimal compared to a one-period coupon, with rate β_H and face value $X^* = V_H - V_L$, which are both lower. Consequently, we do not consider this strategy.

Table 9
Strategies and corresponding payoffs for two sellers offering different duration coupons.

S2	S1: C (Compete)	S1: A (Avoid)
C	$N3^*_{3W}(n_{H1})$, $N3^*_{3F}(n_{H2})$	$NB^*_1(n_{H1})$, $N3^*_3(n_{H2})$
A	$N3^*_3(n_{H1})$, $NB^*_1(n_{H2})$	$NB^*_1(n_{H1})$, $NB^*_1(n_{H2})$

The subscript W in $N3^*_{3W}(n_{H1})$ indicates this is the net benefit for the seller who wins the competition for L-type consumers' redemption in the first period. Consequently, we show that when both S1 and S2 choose to compete with each other, their payoffs are $N3^*_{3W}(n_{H1})$ and $N3^*_{3F}(n_{H2})$, respectively, which are lower than their monopolistic payoffs with optimal three-period coupon $N3^*_3(n_{H1})$ and $N3^*_3(n_{H2})$, correspondingly.

5.2. One seller chooses to compete and the other one chooses to avoid competition

This case is straightforward because head-on competition is absent. One seller will offer a three-period coupon in the first period and the other seller will offer a one-period coupon either in the second or third period.

If S1 chooses to compete and S2 chooses to avoid competition, they will receive corresponding net benefits as $N3^*_3(n_{H1})$ and $NB^*_1(n_{H2})$. Alternatively, if S1 chooses to avoid competition but S2 chooses to compete, they will receive corresponding net benefits as $NB^*_1(n_{H1})$ and $N3^*_3(n_{H2})$.

5.3. Both sellers choose to avoid competition

In this situation, both sellers choose to avoid competition by launching a one-period coupon in different periods. Hence, S1 will receive net benefit $NB^*_1(n_{H1})$ and S2 will receive net benefit $NB^*_1(n_{H2})$. Because both S1 and S2's optimal monopolistic strategy is to offer a three-period coupon in scenario 3, we have $NB^*_1(n_{H1}) < N3^*_3(n_{H1})$ and $NB^*_1(n_{H2}) < N3^*_3(n_{H2})$.

Summarizing the results in the discussions above, we have the strategies and corresponding payoffs presented in Table 9, from which we can examine the equilibria for both sellers.

First, from Table 9, we can conclude the strategy whereby both sellers choose to avoid competition (A, A) will not be the equilibrium, because $NB^*_1(n_{H1}) < N3^*_3(n_{H1})$ and $NB^*_1(n_{H2}) < N3^*_3(n_{H2})$.

Second, if $N3^*_{3W}(n_{H1}) \geq NB^*_1(n_{H1})$ and $N3^*_{3F}(n_{H2}) \geq NB^*_1(n_{H2})$, the strategy whereby both sellers choose to compete (C, C) is the only equilibrium.

Third, if $N3^*_{3W}(n_{H1}) \geq NB^*_1(n_{H1})$ and $N3^*_{3F}(n_{H2}) < NB^*_1(n_{H2})$, the strategy whereby S1 chooses to compete and S2 chooses to avoid competition (C, A) is the only equilibrium.

Fourth, if $N3^*_{3W}(n_{H1}) < NB^*_1(n_{H1})$ and $N3^*_{3F}(n_{H2}) \geq NB^*_1(n_{H2})$, the strategy whereby S1 chooses to avoid competition but S2 chooses to compete (A, C) is the only equilibrium.

Finally, if $N3^*_{3W}(n_{H1}) < NB^*_1(n_{H1})$ and $N3^*_{3F}(n_{H2}) < NB^*_1(n_{H2})$, both strategies (C, A) and (A, C) are equilibrium strategies and neither one is a dominating equilibrium. Consequently, a mixed equilibrium will emerge. Specifically, S1 will compete with probability $\frac{N3^*_3(n_{H2})-NB^*_1(n_{H2})}{N3^*_3(n_{H2})-N3^*_{3F}(n_{H2})}$ but avoid competition with probability $1-\frac{N3^*_3(n_{H2})-NB^*_1(n_{H2})}{N3^*_3(n_{H2})-N3^*_{3F}(n_{H2})}$. And S2 will compete with probability $\frac{N3^*_3(n_{H1})-NB^*_1(n_{H1})}{N3^*_3(n_{H1})-N3^*_{3W}(n_{H1})}$ but avoid competition with probability $1-\frac{N3^*_3(n_{H1})-NB^*_1(n_{H1})}{N3^*_3(n_{H1})-N3^*_{3W}(n_{H1})}$.

6. Conclusion

We develop an analytical model of consumers' coupon redemption that simultaneously considers coupon face value and duration, while incorporating consumers' forgetting and stochastic redemption costs. Our main findings center around (1) optimal face value and duration, (2) coupons as a price-discrimination device, and (3) coupon-redemption patterns.

Optimal coupon face value and duration. Our results provide managers with guidance concerning coupon face value and duration, and provide solutions for optimal face value and payoffs for different duration coupons and redemption patterns, with exogenous and endogenous prices. We provided results for small, medium, and large differences in valuation between L-type and H-type consumers. A small difference in valuations may be consistent with frequently purchased consumable goods like grocery items, whereas a large difference in valuations may be more consistent with more luxury items and novel technologies. Results show a one-period coupon is never optimal when the difference in valuations for the high-value (loyal) and the low-value (non-loyal) consumers, or the coupon face value is larger than the high redemption costs of the high-value consumers, for which longer-duration coupons may still be optimal. Therefore, when managers have little leeway to change prices and a large difference exists in valuations, they should not provide short-duration coupons.

Also, enlarging coupon duration will always result in increased surplus, because consumers can wait and redeem when costs are lower. Overall, when determining the coupon duration, managers need to trade off the gain from increased redemption by low-value consumers for longer-duration coupons, with the loss due to increased redemption by high-value consumers.

Coupons as a price-discrimination device. Endogenously determined prices, together with coupon face value and duration, may lead to a higher price or to a lower price. A higher price with a coupon is straightforward because the seller charges a higher price to offset the cost of the coupon. However, when the difference in valuations of H-type and L-type consumers is large, a retailer may reduce the price, rather than increase the coupon face value and have more H-type consumers redeem coupons. This finding is similar to results by Anderson and Song (2004); however, different from their results, we show that longer-duration coupons may keep H-type consumers from redeeming without having to lower prices. As such, duration plays an important role in the ability to use coupons as a price-discrimination mechanism—importantly, without the need to lower prices.

Coupon-redemption patterns. We also contribute to the literature by proposing a model of rational forward-looking consumers that can explain empirically observed redemption patterns, including monotonic downward trends with a potential spike just before the expiration date, and lower redemption rates for longer coupon durations. These results have important implications for theory building, because they provide new explanations for documented empirical findings, explained solely using theories of irrational (naïve) consumers, such as regret and/or procrastination. Furthermore, they have important managerial implications in their ability to help managers design the duration and face value of coupons.

Finally, to generalize our results, we extend our model to a duopoly with two sellers' competing coupon strategies. We show that if at least one seller's optimal monopolistic strategy is a one-period coupon, they can avoid head-on competition, by alternating the time when they offer the coupon. As a result, each seller will receive their optimal monopolistic net benefit, and results will be consistent with those of the monopoly. However, when using a three-period coupon is optimal for both sellers, either both sellers will compete head-on (resulting in a pure equilibrium) or one seller will use a suboptimal strategy (short-duration coupon) to avoid competition (resulting in a pure or a mixed equilibrium). These results have important implications for competing sellers' coupon strategies, showing equilibrium conditions and conditions when sellers can avoid head-on competition through using different duration strategies. Similar to the results of the monopoly model, we find that coupons can be used as a mechanism to price discriminate after including competition.

Our paper has several limitations that provide promising areas for future research. Although the vast majority of coupons are distributed through traditional channels (not targeted to specific consumers), we do see an increase in targeted coupons and non-traditional coupon strategies (e.g., Jiang, Liu, Shang, Yildirim & Zhang, 2018; Zhou, Cao, Tang & Zhou, 2017). Future research should consider the use of digital coupons (Chiou-Wei & Inman, 2008), which can be customized to individual consumers (Shaffer & Zhang, 2002). How to design and customize coupons to individual consumers is an important area of future research. In doing so, they need to consider issues of fairness, because information that different consumers are treated differently by retailers can easily be shared in online platforms or social networks (Fernandes & Calamote, 2016; Newman, Cinelli, Vorhies & Folse, 2019; Zhou et al., 2017). Future research should generalize the current research by extending the number of consumer types (or a continuous distribution) for product valuation and redemption costs. In addition, we may consider different consumer types for patient versus inpatient consumers, or different types for forgetting to redeem.

Also, empirical research is needed to test the theoretical findings in this study. We proposed a relatively simple framework where a seller offers a single coupon to consumers. Future research should also consider multi-item coupon promotions (Foubert & Gijsbrechts, 2010) and the usage of a series of coupons over time (Nair & Tarasewich, 2003), as well as the combination of coupons and other types of promotions (e.g. Karray, 2011; Martín-Herrán & Sigué, 2015). Finally, future research should extend the optimal coupon design to a manufacturer-retailer supply chain (e.g. Khouja, 2006; Martín-Herrán & Sigué, 2015; Zhang, Popkowski Leszczyc, Qu & Joseph, 2019).

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Supplementary materials

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Appendix A: The seller's optimal strategy for a one-period coupon when price is exogenous

According to Lemma 2, two options are possible whereby a seller should offer a one-period coupon:

Option 1: H-type consumers do not redeem a coupon, and buy at the regular price when costs are high, c_{Hh}, whereas L-type consumers will redeem regardless of whether the redemption cost is c_{Lh} or c_{Ll}. Because $p = V_H$, we obtain $\begin{cases} U_{Ha} = V_H - p + (X - c_{Hh}) < U_{Hb} = V_H - p \Rightarrow X < c_{Hh} \\ U_{La} = V_L - p + (X - c_{Lh}) \geq 0 \;\; \Rightarrow X \geq V_H - V_L + c_{Lh} \end{cases}$. Combining this with $X \geq V_H - V_L$ in Lemma 1, we get $V_H - V_L + c_{Lh} \leq$

$X < c_{Hh}$, which is feasible when $V_H - V_L < c_{Hh} - c_{Lh}$. The net benefit $NB_1 = -\beta_H n_H X + n_L(p - X)$ is maximized at $X^* = V_H - V_L + c_{Lh}$:[9]

$$NB_1^* = n_L(V_L - c_{Lh}) - \beta_H n_H (V_H - V_L + c_{Lh}). \quad \text{(A1)}$$

Option 2: H-type consumers do not redeem the coupon, and buy at the regular price when c_{Hh}, whereas L-type consumers redeem the coupon when the redemption cost is low, c_{Ll}. Because $p = V_H$, we have $\begin{cases} U_{Ha} < U_{Hb} = V_H - p \Rightarrow X < c_{Hh} \\ U_{La} < 0 \Rightarrow X < V_H - V_L + c_{Lh} \end{cases}$. Combining the two conditions for X with the condition $X \geq V_H - V_L$ in Lemma 1, we obtain $V_H - V_L \leq X < Min(c_{Hh}, V_H - V_L + c_{Lh})$, which is feasible when $V_H - V_L < c_{Hh}$. The profit is $NB_1 = \beta_H n_H X + \beta_L n_L (p - X)$, which is maximized when $X^* = V_H - V_L$:

$$NB_1^* = \beta_L n_L V_L - \beta_H n_H (V_H - V_L). \quad \text{(A2)}$$

Appendix B: Seller's optimal strategy for a two- or three-period coupon—price is exogenous

*Two- and three-period coupons (Proposition 5b and Table 4)***:** Here, H-type consumers will postpone coupon redemption when facing a high cost, as stated in Lemma 3. We discuss the couponing strategy according to the different coupons' face values.

B1. When $X \geq Max(V_H - V_L, c_{Hh})$ and $\gamma_H(X) < V_H$

In this case, we know H-type consumers' redemption rates follow a "Mode-2" pattern whereby the total redemption for a two-period coupon will *always* exceed that for a three-period coupon. We next investigate the coupon-redemption behavior of L-type consumers.

Condition $X < c_{Lh}$ cannot exist, because $X \geq Max(V_H - V_L, c_{Hh})$ and $c_{Hh} \geq c_{Lh}$. When $X \geq c_{Lh}$, see Table 2; we know L-type consumers will postpone redemption at cost c_{Lh} when $V_H - V_L \leq X < V_H - V_L + c_{Lh}$ or when $X \geq V_H - V_L + c_{Lh}$ and $\gamma_L(X) < V_H$, and they will redeem immediately when $X \geq V_H - V_L + c_{Lh}$ and $\gamma_L(X) \geq V_H$. According to these three conditions, we have the following:

B1.1. When $V_H - V_L \leq X < V_H - V_L + c_{Lh}$

In this case, L-type consumers' redemption rates follow the Mode-1 pattern. Given that H-type consumers' redemption rates follow a Mode-2 pattern, the benefit of using a three-period coupon is that more L-type but fewer H-type consumers will redeem coupons than when a two-period coupon is issued. Consequently, we conclude a three-period coupon is more profitable than a two-period coupon. The net-benefit objective functions for a three-period coupon is

$$\begin{aligned} N1_3 = &\left[\beta_L + Min(\alpha_L X, 1)(1-\beta_L)\beta_L + Min(\alpha_L X, 1)^2(1-\beta_L)^2\beta_L\right] \\ &\times n_L(V_H - X) - [\beta_H + Min(\alpha_H X, 1(1-\beta_H)\beta_H \\ &+ Min(\alpha_H X, 1)^2(1-\beta_H)^2\big]n_H X. \end{aligned} \quad \text{(B1)}$$

B1.2. When $X \geq V_H - V_L + c_{Lh}$ and $\gamma_L(X) < V_H$

Both L- and H-type consumers' redemptions follow a Mode-2 pattern. The benefit of using a three-period coupon is that fewer H-type consumers will redeem coupons (due to forgetting) than when a two-period coupon is issued, but the cost is that also fewer L-types will redeem. The objective functions for two-period and three-period coupons are

$$\begin{aligned} N2_2 = &[\beta_L + Min(\alpha_L X, 1)(1-\beta_L)]n_L(V_H - X) \\ &- [\beta_H + Min(\alpha_H X, 1)(1-\beta_H)]n_H X \end{aligned} \quad \text{(B2)}$$

$$\begin{aligned} N2_3 = &\left[\beta_L + Min(\alpha_L X, 1)(1-\beta_L)\beta_L + Min(\alpha_L X, 1)^2(1-\beta_L)^2\right] \\ &\times n_L(V_H - X) - [\beta_H + Min(\alpha_H X, 1)(1-\beta_H)\beta_H \\ &+ Min(\alpha_H X, 1)^2(1-\beta_H)^2\big]n_H X \end{aligned} \quad \text{(B3)}$$

B1.3. When $X \geq V_H - V_L + c_{Lh}$ and $\gamma_L(X) \geq V_H$

L-type consumers will not postpone redemption at high cost c_{Lh} and will redeem in period 1, and H-type consumers' redemption follows a Mode-2 pattern. A three-period coupon is more beneficial than a two-period coupon, because fewer H-type consumers but the same number of L-type consumers will redeem coupons than for a two-period coupon. The objective function for a three-period coupon is

$$\begin{aligned} N3_3 = &n_L(V_H - X) - [\beta_H + Min(\alpha_H X, 1)(1-\beta_H)\beta_H \\ &+ Min(\alpha_H X, 1)^2(1-\beta_H)^2\big]n_H X \end{aligned} \quad \text{(B4)}$$

B2. When $V_H - V_L \leq X < c_{Hh}$,

H-type consumers' redemptions follow a Mode-1 pattern. Therefore, the only reason to issue a long-duration coupon is the additional redemption from L-type consumers (also a Mode-1 pattern). The conditions for the Mode-1 pattern for L-type consumers' redemption are $Max(V_H - V_L, c_{Lh}) \leq X < V_H - V_L + c_{Lh}$, or $X < c_{Lh}$, according to Propositions 1 and 3. Otherwise, when L-type consumers follow a Mode-2 pattern, a long-duration coupon will not be beneficial—due to the loss from both L- and H-type consumers' redemptions, which are higher for long-duration coupons than for one-period coupons. When $Max(V_H - V_L, c_{Lh}) \leq X < V_H - V_L + c_{Lh}$, or when $X < c_{Lh}$, the benefit of using a three-period coupon is that more L-type consumers will redeem coupons than when a two-period coupon is issued, but the cost is that also more H-type consumers will redeem. The objective functions for two- and three-period coupons are

$$\begin{aligned} N4_2 = &[\beta_L + Min(\alpha_L X, 1)(1-\beta_L)\beta_L]n_L(V_H - X) \\ &- [\beta_H + Min(\alpha_H X, 1)(1-\beta_H)\beta_H]n_H X \end{aligned} \quad \text{(B5)}$$

$$\begin{aligned} N4_3 = &\left[\beta_L + Min(\alpha_L X, 1)(1-\beta_L)\beta_L + Min(\alpha_L X, 1)^2(1-\beta_L)^2\beta_L\right] \\ &\times n_L(V_H - X) - [\beta_H + Min(\alpha_H X, 1)(1-\beta_H)\beta_H \\ &+ Min(\alpha_H X, 1)^2(1-\beta_H)^2\beta_H\big]n_H X. \end{aligned} \quad \text{(B6)}$$

[9] Subscript 1 in the term NB_1 indicates a one-period duration.

Appendix C

Table A
Seller's net benefit and optimal coupon face value, given different constraints on face value.

Strategy	Constraint on X	Duration	Net benefits	Optimal Face Value
$N1$	$Max(V_H - V_L,\ c_{Hh}) \le X < V_H - V_L + c_{Lh}$, and $\gamma_H(X) < V_H$	3	$N1_3 = [\beta_L + Min(\alpha_L X,\ 1)(1-\beta_L)\beta_L + Min(\alpha_L X,\ 1)^2(1-\beta_L)^2\beta_L]n_L(V_H - X) - [\beta_H + Min(\alpha_H X,\ 1)(1-\beta_H)\beta_H + Min(\alpha_H X,\ 1)^2(1-\beta_H)^2]n_H X$	$X^* = \frac{2[\alpha_L\beta_L n_L(1-\beta_L)+\alpha_H\beta_H n_H(1-\beta_H)-\alpha_L{}^2\beta_L n_L V_H(1-\beta_L)^2]\pm\sqrt{\Delta}}{-6[\alpha_H{}^2 n_H(1-\beta_H)^2+\alpha_L{}^2\beta_L n_L(1-\beta_L)^2]}$ $\Delta = [2[\alpha_L\beta_L n_L(1-\beta_L) + \alpha_H\beta_H n_H(1-\beta_H) - \alpha_L{}^2\beta_L n_L V_H(1-\beta_L)^2]]^2 - 4\ (-3)[\alpha_H{}^2 n_H(1-\beta_H)^2 + \alpha_L{}^2\beta_L n_L(1-\beta_L)^2](-\beta_L n_L - \beta_H n_H + \alpha_L\beta_L n_L V_H - \alpha_L{}^2\beta_L n_L V_H)$
$N2$	$X \ge Max(V_H - V_L + c_{Lh},\ c_{Hh})$ and $Max(\gamma_H(X),\ \gamma_L(X)) < V_H$	2	$N2_2 = [\beta_L + Min(\alpha_L X,\ 1)(1-\beta_L)]n_L(V_H - X) - [\beta_H + Min(\alpha_H X,\ 1)(1-\beta_H)]n_H X$	$X^* = \frac{\beta_L n_L - \alpha_L n_L V_H + \alpha_L\beta_L n_L V_H + \beta_H n_H}{2(\alpha_L\beta_L n_L + \alpha_H n_H\beta_H - \alpha_L n_L - \alpha_H n_H)}$
		3	$N2_3 = [\beta_L + Min(\alpha_L X,\ 1)(1-\beta_L)\beta_L + Min(\alpha_L X,\ 1)^2(1-\beta_L)^2]n_L(V_H - X) - [\beta_H + Min(\alpha_H X,\ 1)(1-\beta_H)\beta_H + Min(\alpha_H X,\ 1)^2(1-\beta_H)^2]n_H X$	$X^* = \frac{-\alpha_H\beta_H + \alpha_H\beta_H{}^2 \pm \sqrt{-3\alpha_H{}^2\beta_H + 7\alpha_H{}^2\beta_H{}^2 - 5\alpha_H{}^2\beta_H{}^3 + \alpha_H{}^2\beta_H{}^4}}{3(\alpha_H{}^2 - 2\alpha_H{}^2\beta_H + \alpha_H{}^2\beta_H{}^2)}$
$N3$	$X \ge Max(V_H - V_L + c_{Lh}, c_{Hh})$ and $\gamma_H(X) < V_H \le \gamma_L(X)$	3	$N3_3 = n_L(V_H - X) - [\beta_H + Min(\alpha_H X,\ 1)(1-\beta_H)\beta_H + Min(\alpha_H X,\ 1)^2(1-\beta_H)^2]n_H X$	Since $N3_3$ decreases with X, the optimal X is the corner solution at the lower bound of the constraint
$N4$	$Max(V_H - V_L, c_{Lh}) \le X < Min(V_H - V_L + c_{Lh},\ c_{Hh})$ or $V_H - V_L \le X < c_{Lh}$	2	$N4_2 = [\beta_L + Min(\alpha_L X,\ 1)(1-\beta_L)\beta_L]n_L(V_H - X) - [\beta_H + Min(\alpha_H X,\ 1)(1-\beta_H)\beta_H]n_H X$	$X^* = \frac{\alpha_L(1-\beta_L)\beta_L n_L V_H - \beta_L n_L - \beta_H n_H}{2\alpha_L(1-\beta_L)\beta_L n_L + 2\alpha_H(1-\beta_H)\beta_H n_H}$
		3	$N4_3 = [\beta_L + Min(\alpha_L X,\ 1)(1-\beta_L)\beta_L + Min(\alpha_L X,\ 1)^2(1-\beta_L)^2\beta_L]n_L(V_H - X) - [\beta_H + Min(\alpha_H X,\ 1)(1-\beta_H)\beta_H + Min(\alpha_H X,\ 1)^2(1-\beta_H)^2\beta_H]n_H X$	$X^* = \frac{2[\alpha_L\beta_L n_L(1-\beta_L)+\alpha_H\beta_H n_H(1-\beta_H)-\alpha_L{}^2\beta_L n_L V_H(1-\beta_L)^2]\pm\sqrt{\Delta}}{-6[\alpha_H{}^2 n_H\beta_H(1-\beta_H)^2+\alpha_L{}^2\beta_L n_L(1-\beta_L)^2]}$ $\Delta = [2[\alpha_L\beta_L n_L(1-\beta_L) + \alpha_H\beta_H n_H(1-\beta_H) - \alpha_L{}^2\beta_L n_L V_H(1-\beta_L)^2]]^2 - 4\ (-3)[\alpha_H{}^2 n_H\beta_H(1-\beta_H)^2 + \alpha_L{}^2\beta_L n_L(1-\beta_L)^2](-\beta_L n_L - \beta_H n_H + \alpha_L\beta_L n_L V_H - \alpha_L{}^2\beta_L n_L V_H)$

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